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A COMPARATIVE STUDY OF BETATRON AND DIRECT INJECTION IN THE ELECTRON SYNCHROTRON PROPOSED FOR THE INSTITUTE OF NUCLEAR PHYSICS, CALCUTTA.

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ABSTRACT. The operation of an electron synchrotron in relation to the types of injection is described briefly and the capture efficiency at the time of start of the synchrotron phase of motion has been calculated. In the two cases the approach of calculation has been different depending on the fact that the motion of electrons towards the end of the betatron phase and the motion at the time of injection from the external injector are different.

The merits of the two types of injection have been discussed.

INTRODUCTION

The synchrotron, which has proved a successful successor to the betatron for acceleration of electrons to very high energies (up to theoretical limit of 2 GEV), is derived from the classical principles of Veksler (1945) and McMillan (1945). It has an a.c. magnet as in the betatron, but the acceleration is provided by an r.f. system as in the cyclotron.

Goward and Barnes (1946) first demonstrated the practical possibility of the electron synchrotron by converting a 4 Mev betatron into an 8 Mev synchrotron simply by increasing the magnet excitation and placing an r.f. resonator outside the donut.

Figures (1) and (2) are block dragrams of the components of a typical machine.

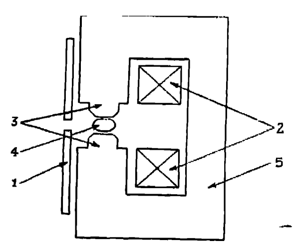

Fig. 1. A "C" Section of synchrotron magnet
1. Betatron flux bar
2. Magnet excitation windings
3. Magnet poles
4. Accleration chamber or donut
5. Magnet flux return path

Injection into a betatron started synchrotron takes place at 50 to 100 KV from an electron gun and electrons are accelerated by betatron action up to relativistic energies. In 'direct' injection electrons of 2 to 3 Mev are injected into

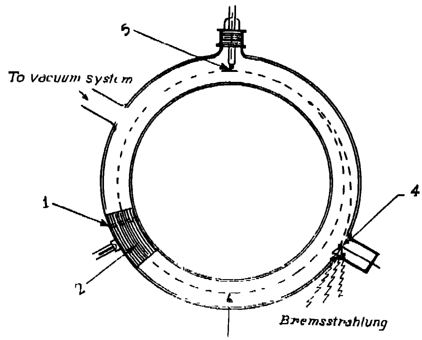

Fig. 2. Details of acceleration chamber or donut
1. Acceleration gap
2 Quarter wave resonator
3. Equilibrium orbit
4. Target
5. Electron gun (injector)

the synchrotron from a linac, Van de Graaff, pulse transformer etc. (Salvini 1955). In the betatron phase of motion, the electrons move in instantaneous orbits which start from the injector position. The instantaneous orbits contract towards the equilibrium orbit r_0 as the energy of the electrons increases along with the increase of the magnetic field. Electrons which are not exactly on their instantaneous circles execute radial and vertical oscillations about the instantaneous circles. The frequency of these radial and vertical oscillations are proportional to $(1-n)^{\frac{1}{2}}$ and $n^{\frac{1}{2}}$ respectively (Kerst 1941) where n is known as the magnetic field index and is given by $-\frac{r}{H}\frac{dH}{dr}$. The amplitudes of these oscillations decrease as the energy increases. Towards the end of the betatron phase of motion, the electron oscillation amplitudes are so small that their effect when the synchrotron phase of motion begins can be neglected.

In the case of 'direct' injection or injection at relativistic energy from an external injector, there is no increase of energy during the time of injection and the instantaneous circles contract due to the increase of magnetic field.

The amplitude of oscillation of electrons which are not on their proper orbits do not decrease but remain as free oscillations. So when the synchrotron action begins there are electrons with free oscillation amplitudes ranging from zero to the clearance between the injector and the stable orbit.

Calculation of efficiency of capture into synchronous motion will depend on the type of motion the electrons are undergoing before the synchrotron phase begins.

GENERAL THEORY

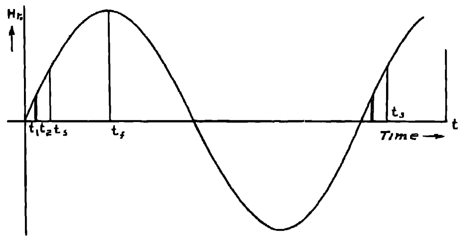

Fig. 3(a). Variation of magnetic field with time and timing of events in betatron injection

Event	Total energy	Time from zero field	Magnetic field at r_0
t_1 (Injection start)	0.56 Mev	9.5 μ Sec	30 gauss
t_2 (Injection stop)	,,	10.22 μ Sec	32 gauss
t_s (R.F.on)	2 Mev	84.9 μ Sec	267 gauss
t_f (R.F.off)	75 Mev	50000 μ Sec	10 K Gau s

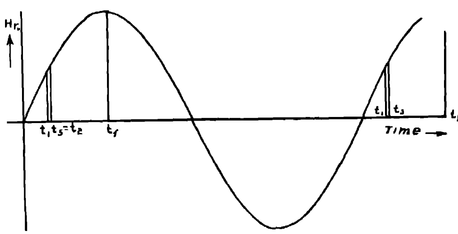

Fig. 3(b). Magnetic field *vs* time and time of event in direct injection

Event	Total energy	Time from zero field	Magnetic field at r_0
t_1 (Injection starts)	2 Mev	82 μ sec	258 gauss
t_2 (Injection Stops R.F. on) $= t_s$	,,	84.7 μ sec	266 gauss
t_f (R.F. off)	75 Mev	5000 μ sec	10, K gauss

Figure 3 (*a*) & (*b*) shows the variation of magnetic field with time at any radius r. The magnetic field varies with radius and time according to the formula

$$H = H_{max}\left(\frac{r}{r_0}\right)^n \sin 2\pi f_m t \tag{1}$$

where H_{max} = maximum magnetic field at the equilibrium orbit radius r_o,
f_m = frequency of magnetic field.

The frequency of the r.f. cycle and the relation of magnetic field with electron energy are given by (Livingston 1954).

$$f_S = \frac{C}{2\pi r_0} \tag{2}$$

$$H = \frac{[E^2 - E^2_o]^{1/2}}{Cer_0} \tag{3}$$

where E_0 = rest energy of the electron.
e = electronic charge in e.s.u.
C = velocity of light

The case of betatron injection.

The spread of radii at the end of the betatron phase and the spread of radii at its beginning are related by

$$\rho_0 = \rho_I\left(\frac{E_I\beta_I}{E_S\beta_S}\right) \tag{4}$$

where ρ_0 = radial spread at the end of betatron phase.
ρ_I = initial spread of radii
E_I = injection energy
E_S = final energy (betatron phase)
β = velocity of electron/velocity of light

The equation of motion of an electron captured into synchronous motion is shown to be an oscillation in phase about a mean positive phase with respect to the r.f. wave (McMillan, 1945, Bohm and Foldy, 1946, Frank 1946). This phase oscillation corresponds to a radial oscillation given by (Goward, 1949).

$$\rho = \frac{\rho_S}{\sqrt{2}} [\cos\theta + \cos\theta_S - (\pi - \theta_S - \theta)\sin\theta_S]^{1/2} \tag{5}$$

ρ_S corresponds to amplitude of oscillation when $\theta_S = 0$, where θ_S is the equilibrium phase angle.

$$\text{Capture efficiency} = \frac{\text{area common to the proper } \theta_S \text{ curve and rectangle}}{\text{area of rectangle.}}$$

(See figure 4)

The case of Direct Injection.

In this case, as these are electrons with all possible amplitudes of free oscillation between 0 and ρ_I, where ρ_I is the clearance between injector and stable orbit, the condition for capture is that the sum free and synchrotron oscillation amplitudes should be less than ρ_I. From this it can be shown that (Persico, 1955)

$$X_M \leqslant \rho_I - \frac{\sigma}{T}\, t \tag{6}$$

where X_M = synchrotron oscillation amplitude

σ = orbit contraction per rotation

T = period of one rotation

t = instant of injection during injection interval.

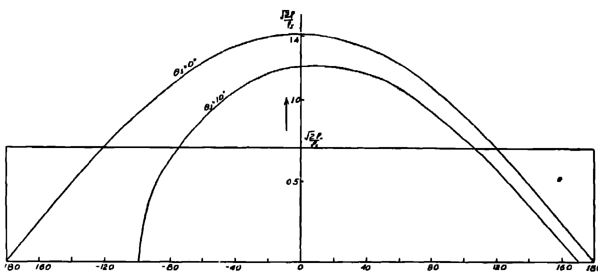

Fig. 4.

X^2_M can be shown to be proportional to a function $\eta(\theta)$ given by

$$\eta(\theta) = [F(\theta) - F(\theta_{max})]$$

where

$$F(\theta) = \frac{\cos\theta}{\sin\theta_s} + \theta$$

Figure (5) is plot of $\eta(\theta)$ against θ. With $\eta(\theta)$ calculated from X_M, as ordinate, a horizontal line is drawn in figure (5). The length of this straight line intercepted by the curve of figure (5) divided by an abscissa length corresponding to 2π gives the captured fraction of electrons injected at time t.

Several such fractions are calculated and plotted in figure (6), where a rectangle is also drawn with ordinate equal to unity (the area of this rectangle corresponds to the number of electrons injected).

$$\text{Capture efficiency} = \frac{\text{area below the curve of figure (6)}}{\text{area of rectangle}}$$

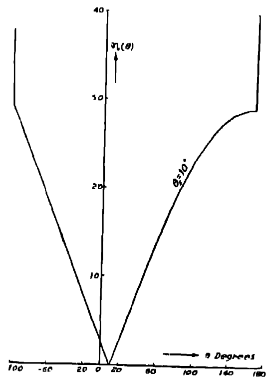

Fig. 5.

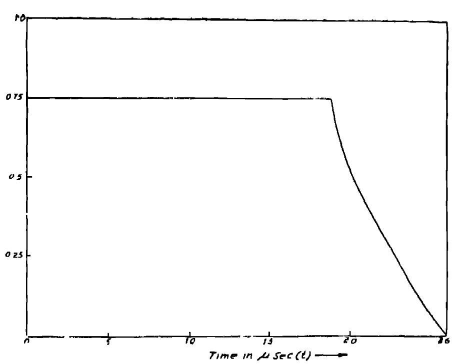

Fig. 6.

Based on these ideas our calculations give the results shown in Tables I, II and III.

TABLE I

Table of common parameters for the electron synchrotron with betatron or direct injection

Parameter	Symbol	Value
Final energy of beam energy	E_f	75 Mev
Peak magnetic field at stable orbit	H_{max}	10^4 gauss
Stable orbit radius	r_0	25 cms.
Peak accelerating voltage	V	725 volts
Radio frequency	f_S	191 Mc/s.
Magnetic field frequency	f_m	50 c/s.
Magnetic field index	n	0.7
Equilibrium phase angle	θ_S	10°
Maximum increase in voltage per rotation	v_0	125 volts
Distance of injector from equilibrium orbit	ρ_I	3 cms.

TABLE II

Additional specifications for betatron injected synchrotron

Parameter	Symbol	Value
Injection energy	E_I	50 K.V. (kinetic)
Starting of time of injection after the magnetic field rises from zero value	t_1	9.5 microseconds
End of injection	t_2	10.22 ,,
Time when betatron stops and R.F. is made on	t_S	84.7 ,,
Electron energy at transition	E_S	2 Mev.
Spread of radii at the end of betatron phase	ρ_0	0.3476 cm.
Size of donut (with elliptical section)	—	4.7 inches half width
	—	3.1 inches half height
The operating pressure	—	1×10^{-5} mm. of Hg.
The max. vertical amplitude due to gas scattering which is attained at four times the injection energy (using Blachman and Courant theory)	—	1 cm.
Fraction of electrons at the end of betatron phase accepted into stable synchronous orbit	—	64.28%

TABLE III

Direct injection parameters

Parameter	Symbol	Value
Injection energy	E_I	2 Mev.
Time of injection start	t_1	82.1 micro-sec.
Time of end of injection end and R. F. on	$t_2=t_S$	84.7 micro-sec.
Donut dimensions—semi major axis	—	4.5″
semi minor axis	—	2.2″
The operating pressure about	—	1×10^{-5} mm. Hg.
At 3.6×10^{-6} mm. of H_g the max. vertical amplitude due to gas scattering	—	0.003676 cm.
Fraction of electrons absorbed into stable orbits after missing the injector	—	64.88%

DISCUSSION

For injection at 50 KV, the magnetic field at the start of injection should be 30 gauss and for injection at 2 Mev the field should be 266 gauss. A higher magnetic field at the time of injection means less difficulty of controlling and shaping the magnetic field.

The theory of gas scattering as worked out by Blackman and Courant (1948) has been shown by Riddiford (1951) to explain approximately the minimum vertical cross section of the donut observed in synchrotrons below which the output of the machine falls rapidly to zero (Elder, Langmuir, Pollock 1948). Using Riddiford's approach we find that the contribution to the oscillation amplitude is negligible in the case of high energy injection. Thus the donut can be operated at higher pressures or the dimensions of the donut can be reduced, as also the cost of the magnet.

The angular spread of an electron beam from a high energy injector is much smaller than that of the conventional betatron injector.

The possibility of electrons missing the injector in the high energy injection case is more than in the betatron injection case as the rate of instantaneous orbit contraction is greater in the former case than in the latter.

The case of betatron injection is simpler, as the additional flux needed is derived by flux bars attached to the poles of the main magnet. In the case of direct injection much additional equipment is needed to bend the high energy beam along the direction of the circular orbits.

High energy injection becomes more and more economical as the maximum energy for which the synchrotron is designed increases as the cost of the injector is nearly the same for all machines.

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